Split quaternions and Carcinogenesis

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Conception of a biologic system

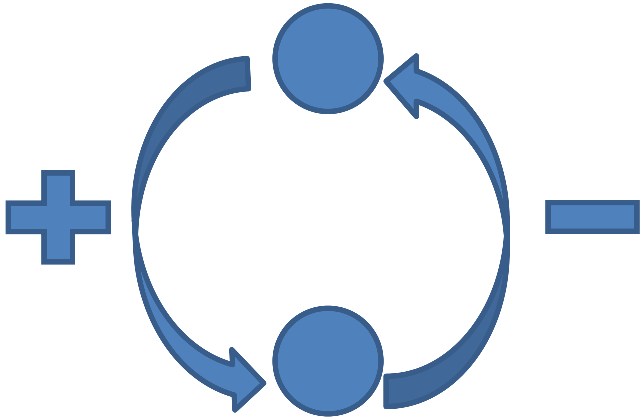
- Examples of BS: cells, tissues, organs, cardiovascular system, respiratory system, endocrine system and so on;
- Common features of BS: set of elements, relations among the elements, specific outcome, steadiness
- Functional structure of BS often is associated with a complex web of elements and links.

Conception of a biologic system

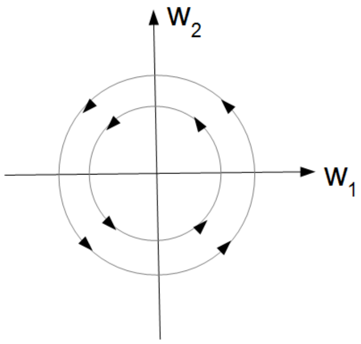
- Abstract system theory was based on self-regulation and stability and included NFB and PFB as functional elements.
- General System (or Biologic System) is defined as a set (collection, constellation) of elements and links comprising a functional structure aimed to a specific outcome.

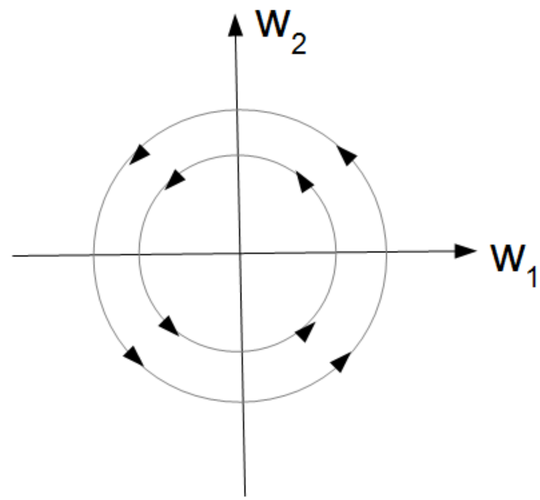
Negative Feedback

- Two-element graph of pituitary-thyroid interactions. (Pituitary gland is at the top)



- Phase curves represent NFB mechanism between two elements





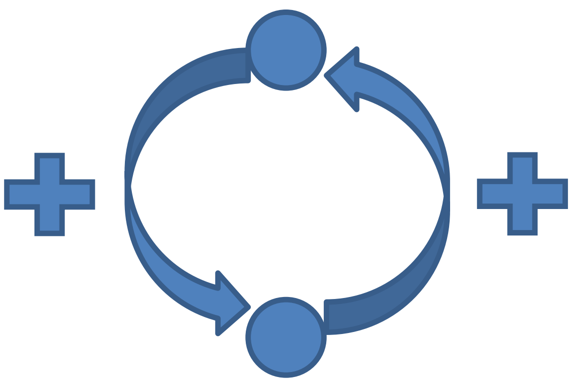
Dynamic representation of NFB

Phase trajectories corresponding to the system

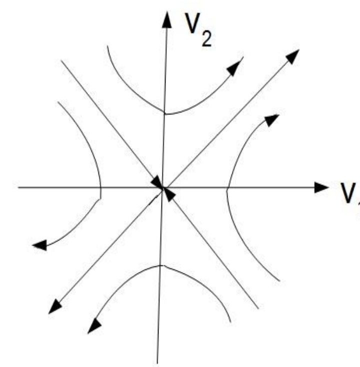
$$\frac{dw}{dt} = S_0 w. \quad S_0 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}, w = (w_1, w_2).$$

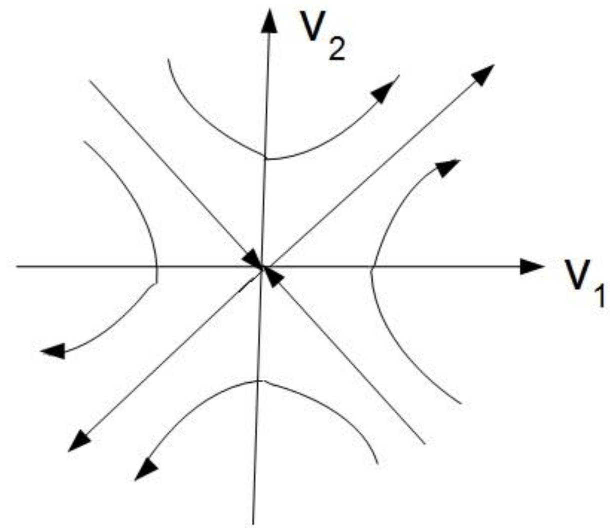
Positive Feedback

- Oxytocin –cervical dilation interactions during the first stage of labour



- PFB curves of the system of two variables



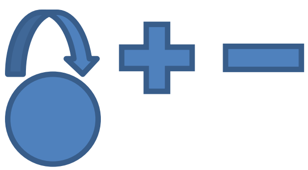


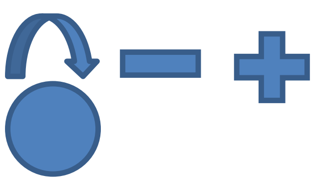
Phase trajectories corresponding to the system

$$\frac{dv}{dt} = S_2 v. \;\; S_2 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, v = (v_1, v_2).$$

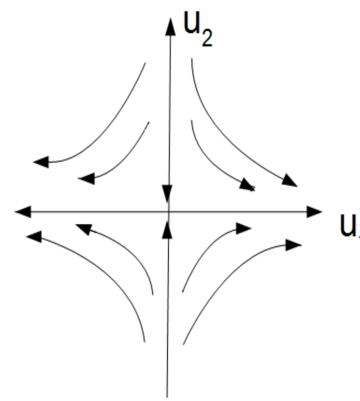
Reciprocal links

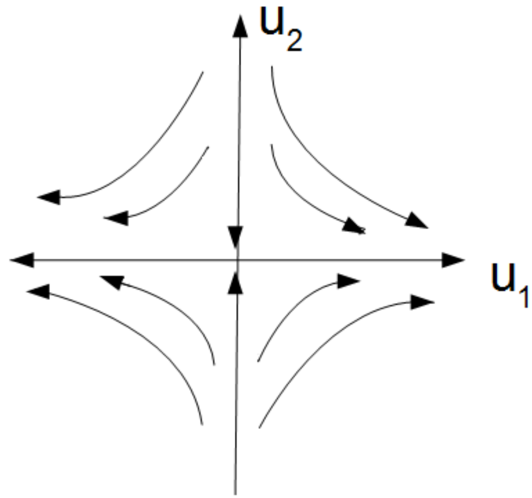
- Clot formation and clot degradation subsystems regulate the viscosity of the blood





- Phase trajectories of reciprocal interactions





Phase trajectories corresponding to the system

$$\frac{du}{dt} = S_1 u. \quad S_1 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}, u = (u_1, u_2)$$

Representation of sl(2,R) in biologic systems

- Negative feedback $S_0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
- Positive feedback $S_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Reciprocal links $S_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

- sl(2,R) = $\left\{ \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \right\}$

General properties of matrices corresponding to NFB, PFB and RL

Matrices representing NFB, PFB and RL are **traceless**, **non-singular** and **linearly independent**

Linear combinations of traceless matrices result in **traceless** matrices

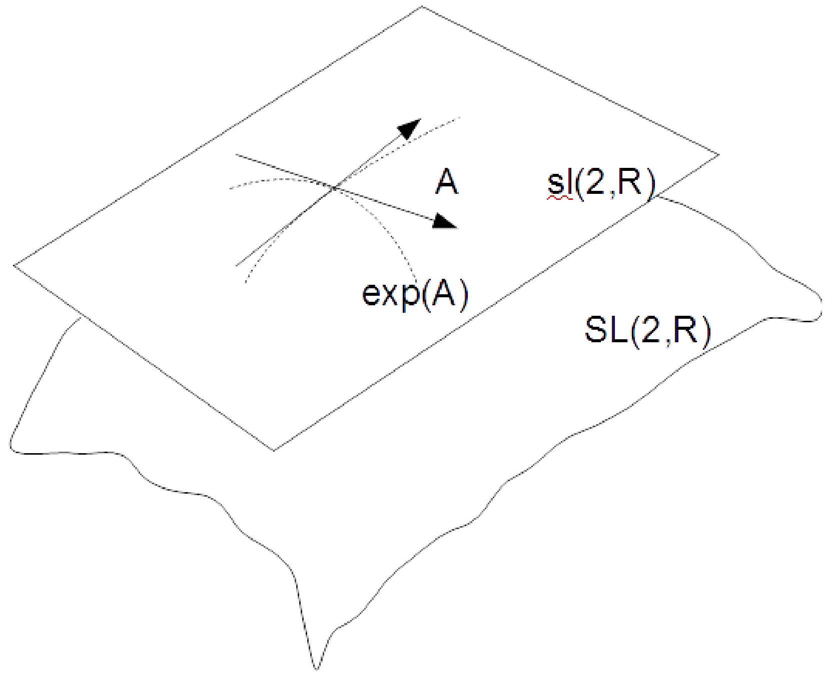
The matrices representing NFB, PFB and RL are identical to the basis matrices of the Lie algebra sl(2,R)

Lie algebra sl(2,R) is a closed structure (an additive group)

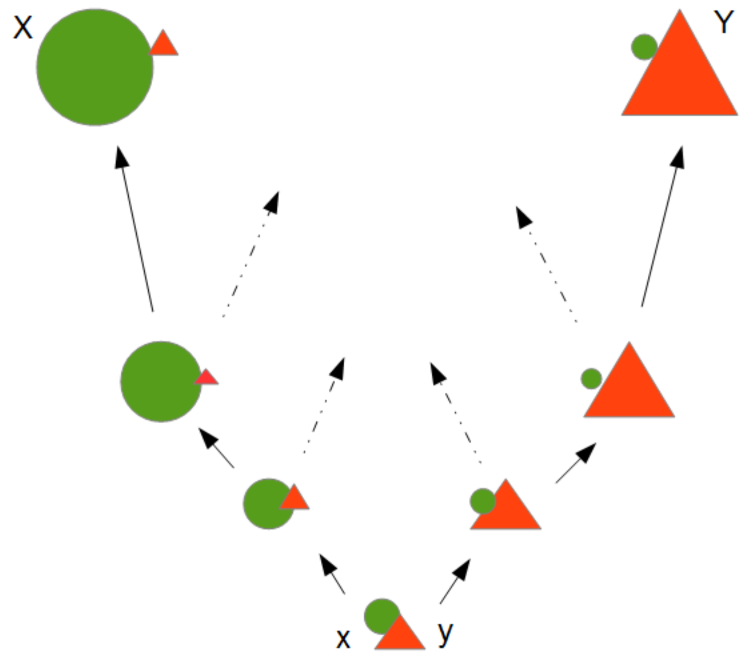
Functional properties of NFB, PFB and RL obtained from matrix representation

- Dynamic processes corresponding to NFB, PFB and RL belong to the same energy level, meaning autonomy and steadiness of regulatory mechanisms.
- Functional integrations of subsystems will also result in steady units.
- NFB, PFB and RL can be considered basis elements of biologic systems
- Functional integration of basis regulatory patterns form a three dimensional space of regulatory elements. "Algebra" of elements form steady functional structure= a System.

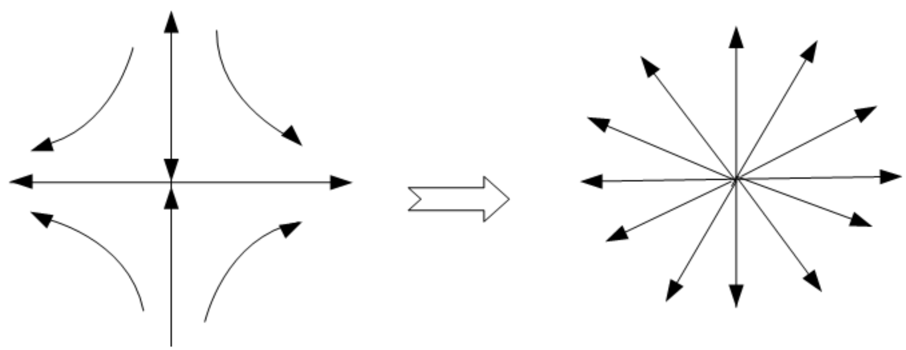
sl(2,R) as a linear approximation of SL(2,R)



The space of transformations SL(2,R)
of biologic variables and its linear approximation-
tangent 3-space sl(2,R)



Schematic representation of phylogenetic splitting of relatively homogeneously distributed characters x and y (green circle and red triangle) at the bottom and clearly distinguishable functional components X and Y at the top, despite the presence of the rudimentary counterparts (small red triangle and green circle).



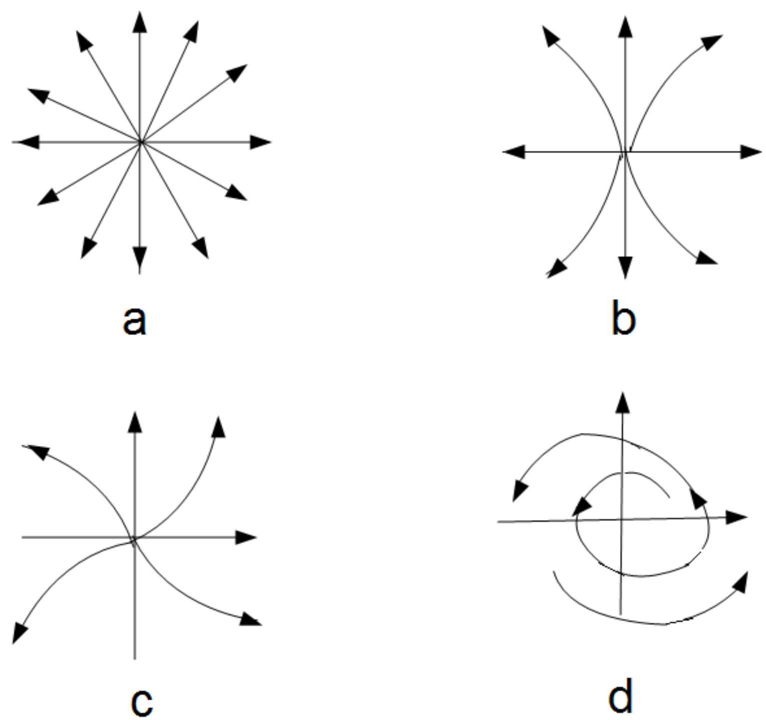
Bifurcation in the system's behaviour is caused by changes in the system's parameters. A saddle transforms to an unsteady node. Two systems are not topologically equivalent- they cannot be transformed to one another smoothly.

Basis elements of a unit split quaternion q={1,i,j,k}

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, i = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, k = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

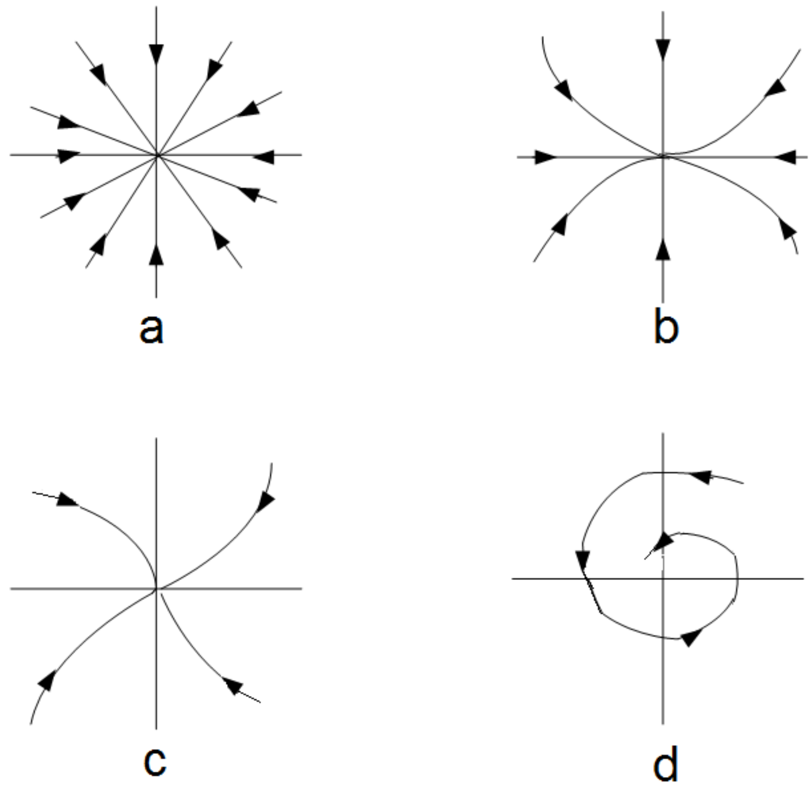
Four classes of dynamic systems exhaust all possible regulatory mechanisms of BS

- Classic description of the steadiness of second order linear systems is given by matrices from q as operators of ODE, for example, $\frac{du}{dt} = Au$, $A \in q$, whose trace, $TrA > 0$, (unsteady systems) or $TrA < 0$(steady systems). These matrices being transformed to the Jordan form have the view
- $J_1 = \begin{pmatrix} \lambda_0 & \\ & \lambda_0 \end{pmatrix}$, $\lambda_0 > 0$ or $\lambda_0 < 0$; $J_2 = \begin{pmatrix} \lambda_0 & \\ & \lambda_1 \end{pmatrix}$, $\lambda_{0,1} > 0$ or $\lambda_{0,1} < 0$, $\lambda_0 \neq \lambda_1$; $J_3 = \begin{pmatrix} \lambda_0 & 1 \\ & \lambda_0 \end{pmatrix}$, $\lambda_0 > 0$; $J_4 = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, a and b are real numbers



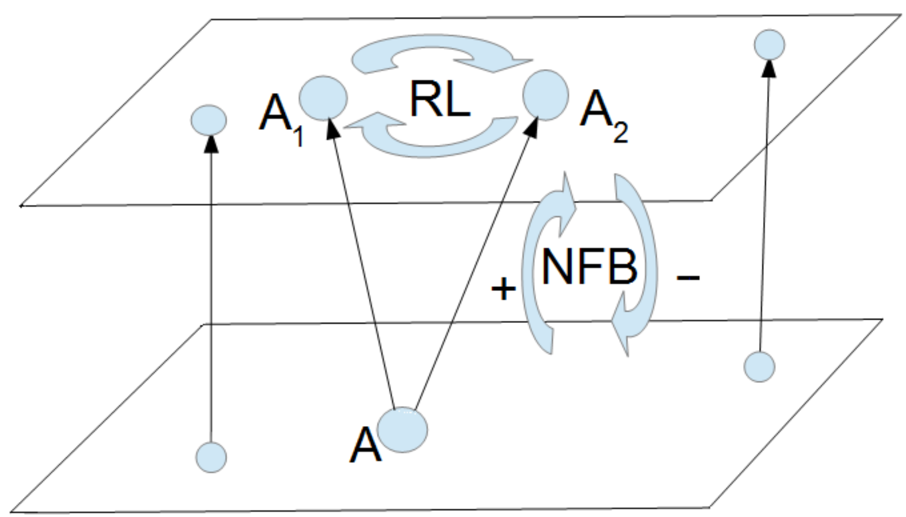
Unsteady systems shown by phase trajectories of ODE $\frac{dx}{dt} = Ax, TrA > 0, detA > 0$:

a) unsteady stellar node corresponding to matrix $A = \begin{pmatrix} p & \\ & p \end{pmatrix}$; b) unsteady node corresponding to $\mathrm{A} = \begin{pmatrix} m & \\ & n \end{pmatrix}$; c) improper node corresponding to $\mathrm{A} = \begin{pmatrix} m & 1 \\ & m \end{pmatrix}$; d) unsteady focus corresponding to $\mathrm{A} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$.



Steady systems shown by phase trajectories of ODE $\frac{dx}{dt} = Ax, TrA < 0, detA > 0$:

a) steady stellar node corresponding to matrix $A = \begin{pmatrix} -p & \\ & -p \end{pmatrix}$; b) steady node corresponding to $A = \begin{pmatrix} -m & \\ & -n \end{pmatrix}$; c) improper node corresponding to $A = \begin{pmatrix} -m & 1 \\ & -m \end{pmatrix}$; d) steady focus corresponding to $A = \begin{pmatrix} -a & b \\ -b & -a \end{pmatrix}$.



Two hierarchical levels of stem cell differentiation. Asymmetric cell division results in splitting of the character A into two daughter characters $A_1$and A_2 linked by reciprocal interactions (RL). Other characters may remain unchanged. Different stages of cell maturation are linked by negative feedback (NFB) sending inhibitory signals to the pool of cell progenitors.